

INTERPLANETARY DIFFUSION COEFFICIENTS FOR COSMIC RAYS

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Information on the cosmic-ray diffusion coefficient, κ , derived from near-Earth observations of the solar modulation of galactic electron fluxes and from the near-Earth power spectra of the interplanetary magnetic field, has been used to study the heliocentric radial dependence of κ and to derive limits on the spatial extent of the solar modulation region.

Representing κ as a separable function of radius r and rigidity, and assuming $\kappa(r) \propto r^n$, we can place a limit on the power law exponent, $n \lesssim 1.2$. The distance of the modulation boundary is a function of n , and, e.g., for $n = 0$, falls into the range of 6-25 AU.

1. Introduction. A knowledge of the radial and rigidity dependences of the interplanetary cosmic-ray diffusion coefficient, κ , is important to relating the observed near-Earth spectra of cosmic rays to their spectra in interstellar space and to studies of the large-scale dynamics of the interplanetary magnetic field.

The diffusion of charged particles in the interplanetary medium is largely governed by pitch-angle scattering caused by the irregular fluctuations of the interplanetary magnetic field. Several investigators have derived equations relating the cosmic-ray diffusion coefficient to the magnetic field power spectrum. We have used these equations to derive the magnitude and rigidity dependence of the local diffusion coefficient from magnetic power spectra measured near 1 AU. Representing the diffusion coefficient as a separable function of radius and rigidity, an assumption consistent with observations of the solar modulation of cosmic-rays, we have independently determined the rigidity dependence $\kappa(R)$ from the cumulative effects of solar modulation of cosmic rays between 1 AU and the modulation boundary. This analysis was based on (a) an estimate of the possible range of interstellar electron spectra as derived from non-thermal radio data, (b) electron spectra measured near Earth, and (c) results of numerical solutions of the cosmic-ray transport equation. Using the magnitude of the power-spectra derived κ as a normalization point for the cosmic-ray derived κ at 1 AU, we can place limits on the radial dependence of κ and make estimates of the size of the modulation region.

2. Diffusion Coefficients from Electron Modulation Study. In Figure 1 we show the locally measured electron spectrum for 1968 and the galactic electron spectrum derived from observations of the galactic non-thermal radio emission (Cummings et al., 1973a). In order to estimate the diffusion coefficient from the electron data we take advantage of the fact that the

logarithm of the ratio of the interstellar and near-Earth electron spectra is a reasonable 1st approximation (Cummings et al., 1973c) to the modulation parameter, ψ , defined by

$$\psi \equiv \int_1^D \frac{Vdr}{\kappa(r,R)} \approx \ln \left[\frac{I(D,R)}{I(1 \text{ AU}, R)} \right] \quad (1)$$

where D is the modulation boundary distance in AU, V is the solar-wind velocity, r is the radial distance from the Sun, R is rigidity, and I represents electron intensity. The modulation parameter calculated from Figure 1 is plotted versus rigidity in Figure 2. We note that the modulation parameter, ψ , is an integral quantity, and therefore the magnitude of the cosmic-ray derived κ at 1 AU depends on the assumed radial dependence of the diffusion coefficient, $\kappa(r)$, and on the modulation-boundary distance, D . As an example, we show in Figure 3 the cosmic-ray derived radial diffusion coefficient $\kappa(R)$, for an assumed radial dependence $\kappa(r) = \text{constant}$, boundary distance $D = 12$ AU, and constant solar-wind velocity $V = 400$ km/sec.

3. Local Diffusion Coefficients from Magnetic Field Power Spectra. The radial diffusion coefficient, κ , at 1 AU is given in terms of the coefficients for diffusion parallel (κ_{\parallel}) and perpendicular (κ_{\perp}) to the average interplanetary magnetic field, B (Jokipii, 1971) by

$$\kappa = 0.45 \kappa_{\parallel} + 0.55 \kappa_{\perp}$$

Assuming the magnetic fluctuations to be one-dimensional waves propagating along the field direction, we have used the perturbation method of Jokipii (1966, 1971) to derive κ_{\parallel} from the power spectrum measured by Quenby and Sear (1971) for the period December 1968 to March 1969. At present, κ_{\perp} cannot be uniquely determined. We show in Figure 3 two power-spectra derived diffusion coefficients corresponding to two different estimates of κ_{\perp} . Curve (1) is computed for the case $\kappa_{\perp} \ll \kappa_{\parallel}$. Curve (2) is based on $\kappa_{\perp} = 4 \times 10^{21}$ cm²/sec, corresponding to the random walk of field lines as estimated by extrapolating the power spectrum to zero frequency (Jokipii and Parker, 1969). Both curves use $V = 400$ km/sec and $B = 5\gamma$.

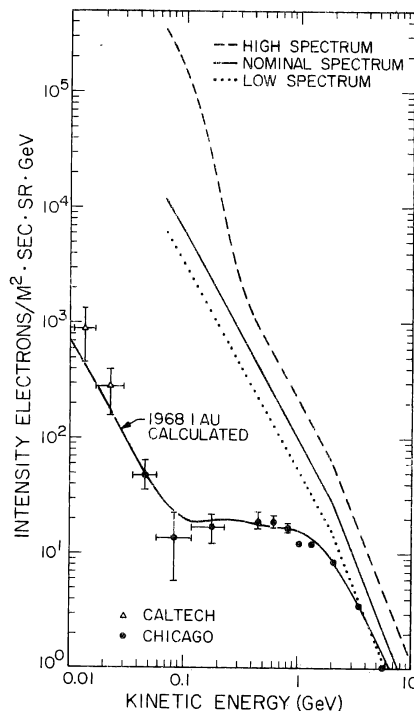


Fig. 1: 1968 cosmic-ray electron spectra near 1 AU and in the interstellar medium. A nominal galactic spectrum is shown as well as two limiting spectra. The solid line through the measured data represents a numerical solution of the transport equation using the nominal interstellar spectrum. (See Cummings et al., 1973b.) Data points:
 Δ Beuermann et al., 1970
 \bullet Schmidt, 1972.

4. Radial Dependence of κ and Distance of the Modulation Boundary.

Although there is only a limited region of overlap, the rigidity dependences of the diffusion coefficients derived from electron modulation and magnetic power spectra are consistent. We recall that the magnitude of the cosmic-ray derived κ at 1 AU depends on the assumed boundary distance D . If we require that the magnitude of the cosmic-ray derived κ agree with that from power spectra at 1 AU, we can place limits on the possible value of the boundary distance. As an illustration, we show in Figure 4 a comparison of the power-spectra derived diffusion coefficients (independent of D) and the cosmic-ray derived κ at 1 AU as a function of D , assuming $\kappa(r) =$ constant. The two magnetic-power-spectra estimates of κ are shown as horizontal bands, corresponding to the 2σ uncertainty of the data. The boundary dependence of the cosmic-ray-derived κ is:

$$\kappa(D, 1 \text{ GV}) = \frac{V \cdot (D-1)}{\psi(1 \text{ AU}, 1 \text{ GV})}$$

and the band in this case results from the uncertainty in our knowledge of ψ , i.e., of the interstellar electron spectrum. The crosshatched areas represent the intersections of the bands. We find that if κ_{\perp} is negligible, boundary distances of 6-15 AU are required for consistency between the two diffusion coefficients. If κ_{\perp} is $4 \times 10^{21} \text{ cm}^2/\text{sec}$, we obtain the boundary range 11-25 AU. If we allow $\kappa(r)$ to be a function of heliocentric radius r , the integral definition of the modulation parameter (equation 1) still determines the magnitude of the diffusion coefficient at 1 AU as a function of boundary distance. By requiring this magnitude to be consistent with that derived from the magnetic power spectra we can calculate the limits on D for any specified radial dependence of κ . As an example, we consider the case $\kappa_{\perp} \ll \kappa_{\parallel}$ and assume κ to be separable function of radius and rigidity, with the radial dependence $\kappa(r) \propto r^n$. We show plots of the limits for the boundary distance D as function of the index n . The horizontal bar at $n = 0$ represents the 6-15 range discussed earlier for κ independent of radius. The minimum boundary distance becomes infinite as n approaches

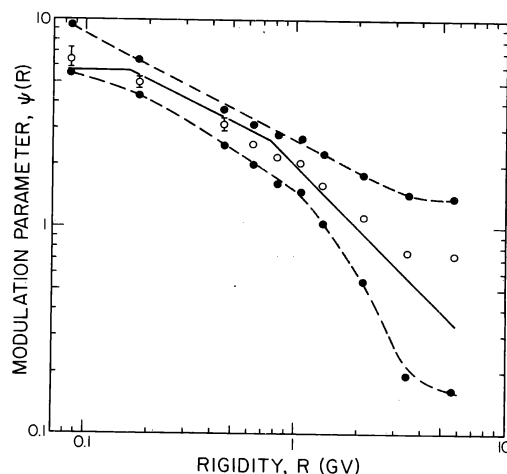


Fig. 2: Modulation parameter ψ vs. rigidity for 1968 electrons. Open circles derived from ratio of nominal galactic spectrum and near-Earth data. Full circles derived from limiting interstellar spectra. The solid line represents the modulation parameter used in deriving the numerical solution shown in Fig. 1.

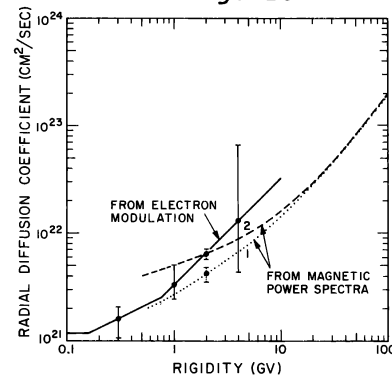


Fig. 3: Radial diffusion coefficient at 1 AU vs. rigidity, derived from electron modulation (solid line) and magnetic power spectra (lines (1) and (2)). Representative error bars of solid line indicate approximate uncertainty according to limiting curves of ψ in Fig. 2. Error bars of curves (1) and (2) reflect 2σ uncertainties of power spectrum.

the limiting value n_c , indicating that for $n > n_c$ we cannot obtain consistency between the diffusion coefficients derived from the magnetic-field power spectrum and from the electron modulation study for any finite value of D . From Figure 5, $n_c = 1.2$ for the case $\kappa_{\perp} \ll \kappa_{\parallel}$. For $\kappa_{\perp} = 4 \times 10^{21}$ cm²/sec, one finds $n_c = 1.1$. Thus, if κ is assumed to increase with r faster than $\sim r^{1.1}$, there is not enough calculated modulation of electrons beyond 1 AU to agree with the observed modulation.

Recently, Jokipii (1973) has calculated the radial dependence of κ for two types of fluctuations: 1) Alfvén waves and 2) frozen-in irregularities. Beyond about 1 AU he finds for Alfvén waves, $\kappa \propto r^0$, and for frozen-in fluctuations, $\kappa \propto 1/r$. From Figure 5, $\kappa_{\perp} \ll \kappa_{\parallel}$, we find that a $1/r$ dependence would imply a boundary range of about 3.3 - 5.5 AU (4.6 - 7.0 AU for $\kappa_{\perp} = 4 \times 10^{21}$ cm²/sec). The r^0 behavior gives the 6 - 15 AU range we derived above.

5. Summary. Assuming the cosmic-ray diffusion coefficient to be a separable function of radius and rigidity, we have found consistency between its cosmic-ray and magnetic-field derived rigidity dependence, $\kappa(R)$. Normalizing the magnitude of the cosmic-ray derived diffusion coefficient $\kappa(r)$ to that from magnetic power spectra near 1 AU, we find the power law exponent in $\kappa(r) \propto r^n$ to be limited to $n \lesssim 1.2$ for the 1968 time period. The cosmic-ray modulation boundary distance, D , is a function of n . For $\kappa(r) = \text{constant}$, i.e., $n = 0$, D falls into the range 6 - 25 AU.

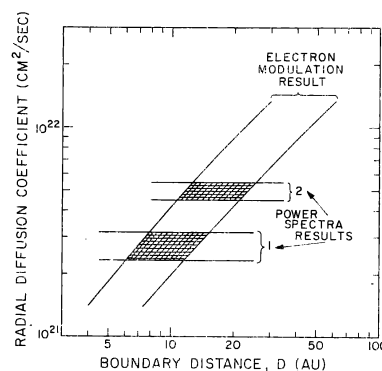


Fig. 4: Radial diffusion coefficients at 1 AU as function of boundary distance D , for electron rigidity $R = 1$ GV.

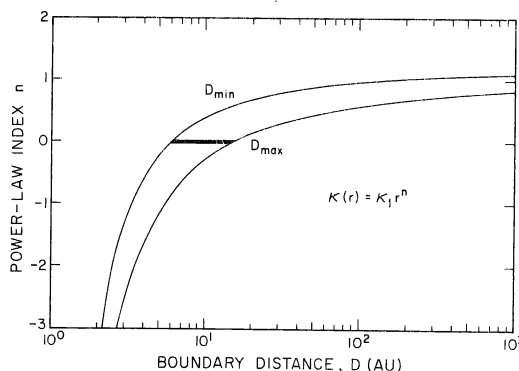


Fig. 5: Modulation-boundary distance, D , as function of n , the power law index of a heliocentric-radius-dependent diffusion coefficient $\kappa(r) = \kappa_1 r^n$.

6. References.

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